

Integrating machine learning and physical models for reconstructing sound speed profiles in mesoscale eddies*

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Abstract Mesoscale eddies significantly affect sound propagation, yet their complex internal sound speed profiles suffer from substantial reconstruction errors, and only few studies specifically addressed this reconstruction. To bridge this gap, we used multi-source satellite data and Argo profiles to identify eddies and build a temperature-salinity-sound speed dataset. Sea surface parameters (temperature, height anomalies, salinity, density) and Argo density serve as inputs for a random forest (RF) algorithm to learn the surface-to-underwater sound speed mapping. Concurrently, a unified eddy dynamic model reconstructs the internal density field. By combining these environmental parameters with the reconstructed density generates eddy sound speed profiles, an integrated PIRF-DEN model that merges machine learning and physical modeling was established. Evaluations demonstrate the superiority of the model. By incorporating density input, the reconstruction accuracy was significantly improved, the mean absolute error (MAE) and root mean square error (RMSE) were reduced to 0.83 and 1.39 m/s, respectively, which is 87.3% and 83.7% less than that of the sEOF-r method. Integration of the eddy model effectively characterized the vertical density structure, whose constraint lowered the overfitting risk and enhanced the accuracy and stability over sEOF-r, sEOF-RF, and RF models. Propagation loss calculations using the reconstructed sound speed showed high correlation (coefficient: 0.77) with measured data, further confirming its reliability.

Keyword: sound speed reconstruction; random forest (RF); unified mesoscale eddy structural model; vertical density

1 INTRODUCTION

Accurate sound speed profiles (SSPs) are critical prior information for analyzing underwater acoustic propagation characteristics. The ocean is a complex nonlinear system encompassing multi-scale dynamic processes such as mesoscale eddies (Chen et al., 2022b), ocean fronts (Chen et al., 2017), and internal waves (Lin and Lynch, 2017), leading to complex spatiotemporal variations in temperature, salinity, and pressure. These variations result in highly uneven spatial distributions of SSPs, making rapid and accurate acquisition of SSPs, which is essential for the activities such as underwater target detection and localization, underwater acoustic

communication, and marine environmental monitoring (Alexander et al., 2016; Wu et al., 2022).

Presently, the most direct method for obtaining SSPs is in-situ observation, which is achieved through direct measurement or indirect calculation using sound velocity profilers (SVPs) or conductivity-temperature-depth (CTD) instruments. In recent years, the successful implementation of the global Argo network has made it a primary source for acquiring underwater SSPs. However, Argo data are also subject to challenges, including significant

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variability in data quality and insufficient coverage in enclosed or semi-enclosed bays and basins (Liu et al., 2024a). The advent of satellite remote sensing technology has led to the availability of real-time, large-scale, high-resolution data on sea surface temperature (SST), and sea surface height anomalies (SSHA). However, the efficacy of these measurements is constrained to the ocean surface or near-surface layers, thus hindering the acquisition of comprehensive subsurface information. Previous studies have demonstrated the efficacy of SST and SSHA in predicting SSPs (Wang et al., 2013; Liu et al., 2017). Therefore, the integration of Argo profiles with satellite-derived sea surface remote sensing data for near-real-time, high-precision SSP reconstruction has emerged as a focal point in research endeavors.

The predominant method for SSP reconstruction is the statistical-regression approach, which uses statistical regression relationships to map surface measurements to the underwater sound speed profile. Theoretically, the SSP can be represented as a function of both space and time. However, this representation requires a large number of parameters. To reduce the dimensionality of SSP reconstruction, the empirical orthogonal function (EOF) method is commonly used. This method performs modal decomposition on SSP data by extracting the principal characteristic modes that capture sound speed variations for reconstruction. LeBlanc and Middleton (1980) demonstrated that using the EOF method as shape functions to describe SSP yields minimal error, allowing SSP to be reconstructed using only the first few coefficients. As a result, the EOF method has become the most widely used approach for underwater sound speed reconstruction. For instance, Chen et al. (2018a) employed the single empirical orthogonal function regression (sEOF-r) method to establish linear regression relationships between SSP's EOF coefficients and sea surface height and temperature anomalies, concluding that sea surface parameters are strongly correlated with only the first-order EOF coefficients. They also analyzed the feasibility and global reconstruction performance of this method. Liu et al. (2023) highlighted that eddy activity significantly influences the SSP distribution. Accordingly, they incorporated eddy kinetic energy (EKE) into the sEOF-r method, optimizing the empirical regression formula to enhance reconstruction accuracy. Although the EOF-based linear framework has some inherent errors, it significantly reduces

computational costs and has even been adopted by the U. S. Navy as part of its operational ocean environment forecasting solution (Fox et al., 2002).

In comparison with conventional linear frameworks, machine learning has been proven more effective in capturing the nonlinear relationships between sea surface environmental parameters and sound speed. Many researchers have combined machine learning algorithms, including neural networks and random forest (RF), with the single empirical orthogonal function (sEOF) method to develop SSP reconstruction models. Through the incorporation of various input parameters, these models have led to a marked enhancement in the accuracy of SSP reconstruction. Park and Kennedy (1996) developed an SSP reconstruction model using a multilayer perceptron neural network and the sEOF method, with SST, temporal information, and acoustic multipath travel times as the inputs. Recognizing the complexity of shallow waters and the stability of deep waters, Liu et al. (2020) applied an adaptive approach to determine the optimal EOF mode order, thereby enhancing SSP reconstruction accuracy. Li et al. (2021) applied a self-organizing maps (SOM) neural network combined with the sEOF method to reconstruct SSPs in the South China Sea by incorporating SST anomalies, SSHA, and geolocation data as inputs. Building upon the foundational work of Li et al. (2021), Ou et al. (2022) have refined SSP estimates by leveraging observed sea surface data to enhance the quality of satellite remote sensing data. Similarly, Li et al. (2022) adopted this method while integrating sea surface sound speed as an additional input, successfully reconstructing SSPs in the southeastern Indian Ocean and further validating the effectiveness of the SOM approach.

Recent advancements in machine learning have led to the development of increasingly sophisticated frameworks. Zhao et al. (2024) employed the sEOF method in conjunction with a long short-term memory (LSTM) network to reconstruct SSPs in the South China Sea, using sea temperature anomalies and SSHA as inputs. Similarly, Liu et al. (2024b) integrated the sEOF method with generalized regression neural networks (GRNN) and thereby established a nonlinear mapping between sea temperature anomalies, sea level anomalies, EKE, and SSPs in the Luzon Strait. In addition to these advancements, Feng et al. (2024) incorporated shallow water temperature, flow field data, and geolocation into the model framework, and then

compared the reconstruction performance of various machine learning algorithms within the same SSP reconstruction framework. Li et al. (2025) proposed a fast SSP reconstruction method (3dEOF-RF) based on the three-dimensional EOF method and the RF algorithm by using SST and sea surface height as model inputs, which significantly improved both reconstruction accuracy and efficiency.

Overall, the hybrid model combining sEOF with machine learning demonstrates superior performance in capturing the nonlinear surface-to-underwater sound speed relationship, featuring an optimized structure with enhanced regional applicability, scalability, and significantly improved reconstruction accuracy and efficiency. However, it exhibits notable limitations: strong regional dependency that hinders generalization across diverse marine environments, high sensitivity to training data quality and quantity (potentially leading to overfitting/underfitting), and limited interpretability of its internal decision-making processes, which may restrict its utility in physics-driven applications.

It is noteworthy that Liu et al. (2023), after reconstructing global SSPs, identified regions with significantly larger errors concentrated in the Kuroshio Extension region of the northwestern Pacific, the Gulf Stream region of the North Atlantic, the Southern Hemisphere's Antarctic Circumpolar Current region, the Brazil Current region, and the East Australian Current region—all areas with high eddy occurrence frequencies. Similarly, Li et al. (2025) employed the 3dEOF-RF method to reconstruct underwater sound speed in

the northwestern Pacific, finding that reconstruction errors within eddies were persistently elevated (as shown in the red-boxed regions in Fig.1). In these regions, the MAE and RMSE of reconstructed sound speed within eddies exceeded 8 m/s, which is more than 50% higher than the average error in the study area.

The findings indicate that the accuracy of sound speed reconstruction within eddies remains low. Possible reasons for this include the following: (1) eddies induce vertical seawater movement, leading to strong sound speed disturbances and complex vertical SSP structures that are difficult to model; (2) the limited availability of SSP samples within eddies, combined with the exclusive reliance on sea surface remote sensing data for modeling, prevents a comprehensive understanding of the internal SSP structure, thereby restricting reconstruction accuracy; (3) the EOF method, which prioritizes primary modes for modeling, has been observed to overlook certain sound speed details, thereby reducing reconstruction precision.

It is widely acknowledged that mesoscale eddies exert a substantial influence on sound propagation (Zhu et al., 2021; Chen et al., 2022a; Li et al., 2024), and the accuracy of sound speed within eddies directly affects the refinement of mesoscale eddy sound field analysis. However, the majority of SSP reconstruction studies have centered on global oceans or specific regions, with a paucity of research specifically targeting SSP reconstruction within mesoscale eddies. To address this knowledge gap, we extracted all Argo profiles located within

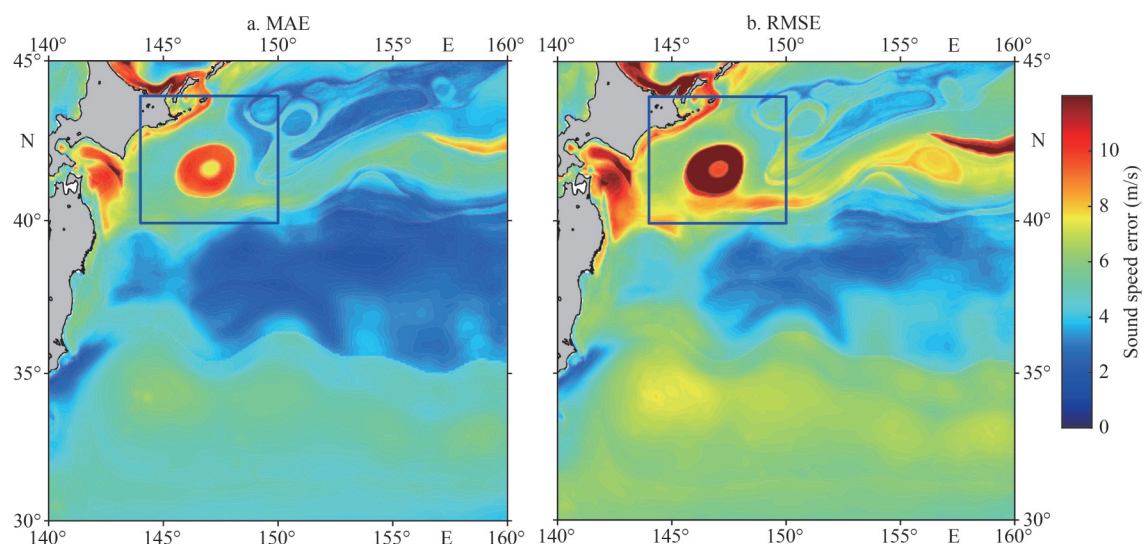


Fig.1 Mean absolute error (MAE) (a) and root mean square error (RMSE) (b) of SSP reconstruction in the northwestern Pacific using the 3dEOF-RF method (after Li et al., 2025)

eddies in the northwestern Pacific from 1997 to 2024. Combining sea surface satellite remote sensing data with a unified mesoscale eddy structural model, we proposed a model that integrates machine learning with physical modeling—the PIRF-DEN model—designed specifically to reconstruct sound speed within mesoscale eddies. The model's primary objective is to enhance the precision of underwater SSP reconstruction within eddies.

2 DATA AND METHOD

2.1 Study area

The Kuroshio Extension region (130°E–180°E, 25°N–45°N) is a region of strong currents where the Kuroshio flows northeastward after passing south of Japan. Upon separating from the Japanese coastline and entering the North Pacific, the Kuroshio experiences increased flow due to the addition of recirculating water and heightened instability. This area is considered one of the most active regions for oceanic eddies (White and McCreary, 1976; Yasuda et al., 1992). The SSP structure within this area is characterized by its complexity, which leads to significant errors in sound speed reconstruction. For the present study, the northwestern Pacific Kuroshio Extension region was selected (Fig.2) to conduct targeted SSP reconstruction experiments within mesoscale eddies. The experimental dataset includes Argo profile data and satellite remote sensing data.

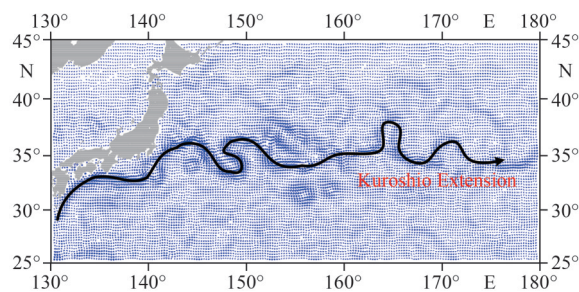


Fig.2 Schematic of the Kuroshio Extension region

The black line represents the axis of the Kuroshio Extension current.

The Argo profiles were obtained from the Argo Real-Time Data Center (<http://www.argo.org.cn/>). The sea surface satellite remote sensing data, including SST, sea level anomaly (SLA), sea surface salinity (SSS), sea surface density (SSD), as well as climatological temperature-salinity data (ARMOR 3D) for density reconstruction within eddies, were provided by the CMEMS platform (<https://marine.copernicus.eu/>). Detailed information on these datasets was summarized in Table 1.

Density values needed for modeling were calculated using the Seawater toolkit, while sound speed was derived from the Argo profiles using the UNESCO algorithm (Chen and Millero, 1977), with the specific empirical formulas detailed in the work of Wong and Zhu (1995).

2.2 Random forest algorithm

RF is an ensemble learning method based on decision trees that significantly improves prediction accuracy and stability by constructing multiple decision trees and integrating results through averaging. This algorithm has demonstrated notable advantages in marine environmental parameter reconstruction (Bao, 2021; Yan, 2021; Wang et al., 2025): (1) strong nonlinear modeling capability to accurately characterize complex relationships between sea surface parameters and underwater thermohaline profiles; (2) ensemble learning mechanism effectively suppresses overfitting through multi-tree voting, significantly enhancing model generalization in unsampled regions; (3) robustness against input data noise and missing values, making it particularly suitable for handling incomplete marine observation data; (4) built-in feature importance evaluation that provides quantitative identification of key drivers influencing thermohaline distribution. Compared to traditional empirical models and single machine learning algorithms, RF exhibits superior performance in computational efficiency, inversion accuracy, and model interpretability. Building upon RF's successful

Table 1 Data type

Data source	Element	Time scope	Temporal resolution	Spatial resolution
Argo	Temperature and salinity	January 1997–March 2024	1 d	—
DUACS	SLA		1 d	0.25°×0.25°
GHRSSST	OISST		1 d	0.05°×0.05°
CNR	SSS and SSD		1 d	0.125°×0.125°
ARMOR 3D	Temperature and salinity		1 month	0.25°×0.25°

— means no data.

applications in thermohaline element reconstruction and its methodological strengths, this study innovatively introduces RF to mesoscale eddy sound speed profile reconstruction, and established an RF-based sound speed profile reconstruction model.

2.3 Current methodology for sound speed profile reconstruction

Current SSP reconstruction models based on sea surface environmental parameters to establish “surface-to-underwater” sound speed mapping relationships can be primarily categorized into two types: the sEOF-based sound speed reconstruction method, and the machine learning-based depth-wise sound speed reconstruction method. While the EOF method demonstrates stable performance in localized regions, machine learning approaches exhibit greater flexibility and accuracy on a global scale. Integrating these two SSP reconstruction paradigms, this study develops three distinct models for reconstructing SSP within mesoscale eddies (sEOF-r, sEOF-RF, and RF methods), which are subsequently compared with the proposed PIRF-DEN model to validate its effectiveness and superiority. The root mean square error (RMSE) and the mean absolute error (MAE) were employed as metrics to quantify the error between the estimated sound speed values and the test samples.

2.3.1 sEOF-r method

The primary goal of the EOF method is to separate the temporal and spatial variable functions in the SSP, describing the spatiotemporal variations of the SSP using as few modal basis functions as possible. Integrate all SSPs within mesoscale eddy into a sound speed profile matrix $C_{M \times N}$, where M represents the number of vertical layers in the SSP, and N represents the number of SSPs within the region. By performing EOF decomposition on the SSP matrix $C_{M \times N}$, this matrix can be approximately expressed as:

$$C_{M \times N} = C_0(M) + \sum_{k=1}^K \alpha_k f_k, \quad (1)$$

where K is the number of EOFs selected for computation, α_k is the empirical orthogonal coefficient, f_k is the EOF base function. Its background profile, denoted $C_0(M)$, is defined as the row-wise mean of matrix C . This uniform C_0 is employed in all subsequent reconstructions to ensure a consistent background field and to facilitate comparative analysis. The selection of the order

depends on the variance contribution rate of each mode, with the cumulative variance contribution rate of the first K eigenvectors is given by Eq.2:

$$Q = \frac{\sum_{i=1}^K \lambda_i}{\sum_{m=1}^M \lambda_m}. \quad (2)$$

Research has shown that when the cumulative variance contribution rate of the first three EOFs (Q) is $\geq 88\%$, reliable reconstruction results can be obtained (Liu, 2020).

In the traditional sEOF-r method, the EOF coefficients are typically computed for each location's SSP matrix based on Eq.1. Following the work of Chen et al. (2018a), the relationships between SST and SLA with SSP are significant. Therefore, we constructed a univariate linear regression between the sea surface parameters (SST and SLA) and the temporal coefficients α_k of the EOFs. The least squares estimation of the fitting coefficients was obtained, and the linear regression relationship is expressed by Eq.3:

$$\alpha_k = b_{0k} + b_{1k} \text{SST} + b_{2k} \text{SLA} + b_{3k} \text{SST} \times \text{SLA}. \quad (3)$$

2.3.2 sEOF-RF method

It is well known that the ocean system is complex and nonlinear, and using a linear framework to reconstruct the SSP inevitably introduces errors. To address this, the sEOF-r method is extended by leveraging machine learning to capture nonlinear relationships. Specifically, the RF algorithm is employed to model the nonlinear relationship between the EOF temporal coefficients α_k and SST and SLA, forming the sEOF-RF method. The specific reconstruction process is shown in Fig.3.

2.3.3 RF method

To further evaluate the enhancement achieved by incorporating reconstructed density into SSP reconstruction within eddies, this study designs an RF model that excludes the vertical density field of seawater. Instead, it directly uses sea surface environmental data (SST, SLA, SSS, and SSD) as inputs for SSP reconstruction based on RF. The sole difference between this approach and the PIRF-DEN model lies in the absence of density as an input, all other procedures remain identical. Consequently, the detailed modeling steps are not repeated here.

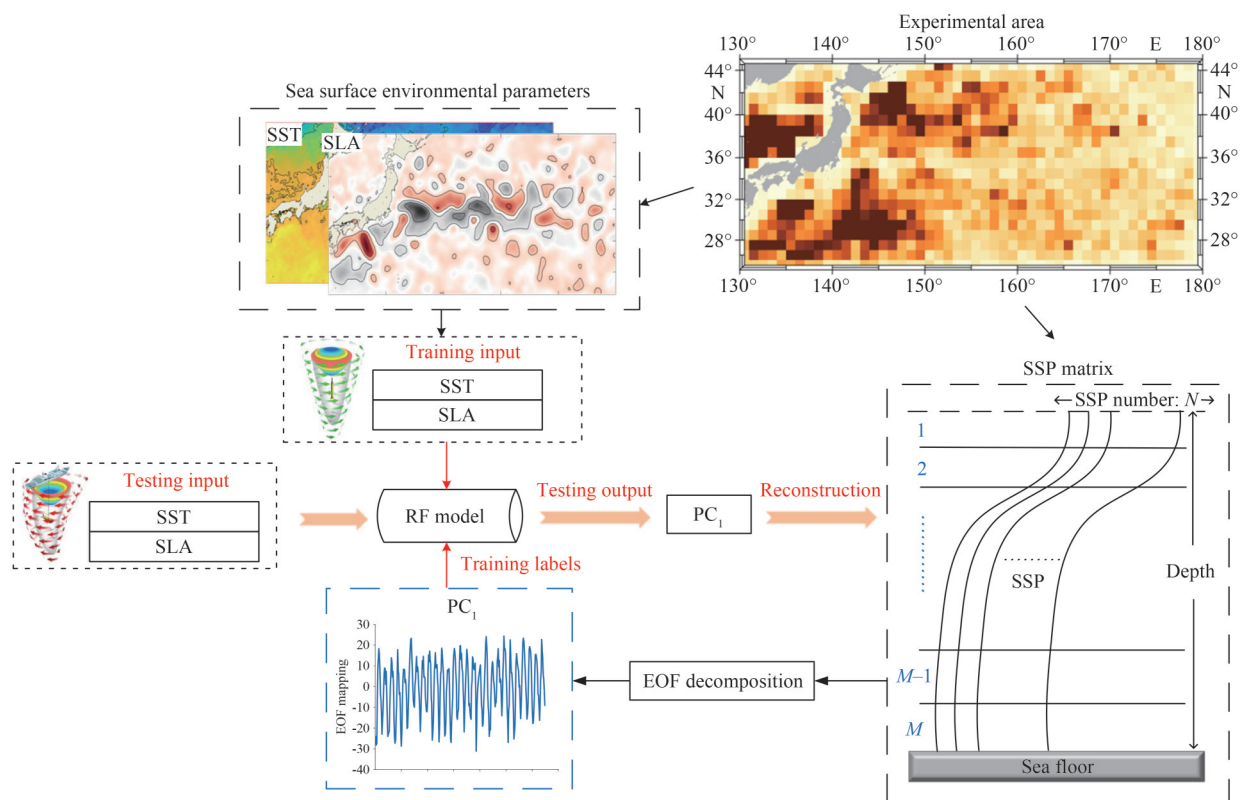


Fig.3 The technical route of sEOF-RF

SSP: sound speed profile; SST: sea surface temperature; SLA: sea level anomaly; RF: random forest; EOF: empirical orthogonal function; PC_1 : the first EOF mode; M : vertical layer count.

3 PIRF-DEN MODEL

The temperature and salinity characteristics inside and outside mesoscale eddies are distinct, and the vertical structure of the sound field is highly complex. Relying solely on sea surface environmental parameters limits the accuracy of SSP reconstruction. The theoretical formula for sound speed is as follows:

$$c = \frac{1}{\sqrt{\rho\beta}}, \quad (4)$$

where ρ is seawater density, β is the adiabatic compressibility coefficient of seawater. This equation clearly shows that sound speed is closely related to seawater density.

Previous SSP reconstruction experiments conducted by our team in the northwestern Pacific demonstrated that incorporating density as vertical information into the SSP reconstruction model could significantly improve the accuracy. The MAE of the reconstructed sound speed decreased from 4.04 to 1.17 m/s, while the RMSE decreased from 4.95 to 1.54 m/s. Thus, the key challenge in developing a

mesoscale eddy sound speed reconstruction model is to obtain the density field within the eddy.

Presently, the availability of real-time measurement data within eddies is limited, and the acquisition of a substantial number of density profiles inside eddies needs long-term observation, which is costly. The density field within eddies derived from reanalysis data exhibits significant discrepancies with in-situ observations (Chen et al., 2018b; Wang et al., 2022), and the direct use of these data would exacerbate reconstruction errors. The density field reconstructed based on surface quasi-geostrophic (SQG) theory resembles only the first baroclinic mode (Scott and Wang, 2005; Lapeyre and Klein, 2006) and poorly characterizes other baroclinic modes, leading to errors in the vertical structure of the reconstructed field compared to the real ocean environment. In contrast, the unified mesoscale eddy structural model by Zhang (2014) proposes that mesoscale eddies within a specific region have analogous horizontal and vertical structures. This model helps the reliable reconstruction of 3D density anomaly fields within the eddy by using a single observed profile, thereby

addressing the challenge of acquiring in-situ density profiles within eddies.

In this study, we used Argo density data from within the eddy and sea surface environmental parameters (SST, SLA, SSS, and SSD) as inputs for SSP reconstruction. We employed an RF algorithm to establish the mapping relationship from sea surface environmental parameters to underwater sound speed within the eddy. Using a solitary observed profile from the eddy, we implemented the unified mesoscale eddy structural model to reconstruct the density field within the eddy, integrated the sea surface environmental parameters with the reconstructed eddy density, and output the eddy's SSP. This approach established a mesoscale eddy SSP reconstruction model that integrates machine learning and physical models, being referred to as the PIRF-DEN model. The name PIRF-DEN clearly defines the model's three core features: "PI" stands for "Physics-Informed," signifying the integration of the unified mesoscale eddy structural model as a physical constraint; "RF" indicates that the model uses the random forest algorithm as its fundamental framework; and "DEN" denotes "Density", highlighting the novel incorporation of the vertical density profile as a critical input, which works in concert with sea surface parameters to drive the model. The subsequent sections delineate the density reconstruction process based on the unified mesoscale eddy structural model and the technical route of the PIRF-DEN model.

3.1 Density reconstruction based on the unified mesoscale eddy structural model

The comprehensive analysis method by Zhang (2014), which was developed based on extensive Argo observational data collected within eddies, has demonstrated that the mesoscale eddy pressure anomaly field exhibits independent horizontal and vertical structures. Specifically, the shape of the pressure anomaly curves at different depths remains essentially unchanged with depth, while the pressure field itself is derived from the integral of density with respect to depth. Therefore, the reconstruction of the density field is a key challenge addressed by the unified mesoscale eddy structural model. This finding also indirectly corroborates the notion that density plays a pivotal role in delineating the vertical structure of mesoscale eddies. According to this theoretical framework, the mesoscale eddy pressure anomaly field can be decomposed into the product of its horizontal and vertical structures:

$$p'(r, z) = p_0 \cdot R \left(\frac{r}{R_0} \right) \cdot H(z), \quad (5)$$

where p_0 is the pressure anomaly at the sea surface at the eddy center, R_0 is the eddy radius, (r/R_0) is the normalized mesoscale eddy radius, $R(r/R_0)$ and $H(z)$ are the radial and vertical structure functions of the normalized mesoscale eddy pressure anomaly field, respectively.

The pressure anomaly at the sea surface at the eddy center, p_0 , can be calculated using satellite observations of the height anomaly η_0 and the mesoscale eddy center density ρ_0 :

$$p_0 = \rho_0 g \eta_0. \quad (6)$$

Using the hydrostatic equilibrium equation, the density anomaly field structure can be derived as:

$$\rho'(r, z) = -\frac{1}{g} \frac{\partial p'}{\partial z} = -\rho_0 \eta_0 \cdot R \left(\frac{r}{R_0} \right) \frac{dH(z)}{dz}. \quad (7)$$

Similarly, the density anomaly can be expressed as the product of horizontal and vertical structure functions:

$$\rho'(r, z) = J(z) \cdot \eta(r), \quad (8)$$

where the horizontal structure function is:

$$\eta(r) = \eta_0 \cdot \left[1 - \frac{1}{2} \left(\frac{r}{R_0} \right)^2 \right] \cdot \exp \left[-\frac{1}{2} \left(\frac{r}{R_0} \right)^2 \right], \quad (9)$$

and the vertical structure function is:

$$J(z) = -\rho_0 \frac{dH(z)}{dz}. \quad (10)$$

Once the density anomaly is obtained, the mesoscale eddy's density field structure can be derived by adding the climate mean density field $\bar{\rho}(z)$:

$$\rho(r, z) = \rho'(r, z) + \bar{\rho}(z). \quad (11)$$

Based on these principles, once the density structure at a specific point within a mesoscale eddy is obtained, the three-dimensional density field can be reconstructed using sea surface height anomaly data and regional climate mean data. The specific steps are as follows:

(1) Use a mesoscale eddy identification algorithm to obtain the eddy center, radius, and other relevant information.

(2) Extract the density profile $\rho_1(r, z)$ at a specific point within the eddy, subtract the regional climate mean density, and calculate the density anomaly $\rho_1'(r, z)$.

(3) Using Eq.10, decompose the density anomaly at the observed point $\rho_1'(r_1, z_1)$: $\rho_1'(r_1, z_1) = \eta(r_1) \cdot J(z_1)$. The density anomaly at the target reconstruction point (r_2, z_2) is denoted as $\rho_2'(r_2, z_2)$, and the ratio between the reconstructed and observed points is:

$$\frac{\rho_2'(r_2, z_2)}{\rho_1'(r_1, z_1)} = \frac{\eta(r_2) \cdot J(z_2)}{\eta(r_1) \cdot J(z_1)}. \quad (12)$$

According to the unified mesoscale eddy structural model, $J(z_1)$ and $J(z_2)$ within the same eddy are approximately equal, i.e., $J(z_2) \approx J(z_1)$. Substituting $\eta(r)$ into Eq.12, we obtain:

$$\frac{\rho_2'(r_2, z_2)}{\rho_1'(r_1, z_1)} = \frac{\eta_0 \cdot \left[1 - \frac{1}{2} \left(\frac{r_2}{R_0} \right)^2 \right] \cdot \exp \left[-\frac{1}{2} \left(\frac{r_2}{R_0} \right)^2 \right]}{\eta_0 \cdot \left[1 - \frac{1}{2} \left(\frac{r_1}{R_0} \right)^2 \right] \cdot \exp \left[-\frac{1}{2} \left(\frac{r_1}{R_0} \right)^2 \right]}. \quad (13)$$

After simplifying, we obtain:

$$\rho_2'(r_2, z_2) = \rho_1'(r_1, z_1) \cdot \frac{\left[1 - \frac{1}{2} \left(\frac{r_2}{R_0} \right)^2 \right]}{\left[1 - \frac{1}{2} \left(\frac{r_1}{R_0} \right)^2 \right]} \times \frac{1}{\exp \left[\frac{1}{2} \left(\frac{r_2}{R_0} \right)^2 - \frac{1}{2} \left(\frac{r_1}{R_0} \right)^2 \right]}. \quad (14)$$

Thus, the density anomaly at the reconstructed point $\rho_2'(r_2, z_2)$ can be determined. Finally, by adding the density anomaly field to the climate mean density field, the mesoscale eddy density field is fully reconstructed.

3.2 Technical route of the PIRF-DEN model

First, we constructed a comprehensive dataset of thermohaline, density, and sound speed characteristics within mesoscale eddies using Argo profiling data, then combined satellite-derived sea surface parameters (SST, SLA, SSS, and SSD) with Argo density profiles as input features to establish the “surface-to-underwater” sound speed mapping relationship using RF algorithm. Furthermore, we reconstructed the three-dimensional density field within eddies by applying a unified eddy structure model. The reconstructed density field, along with sea surface environmental parameters, was then incorporated into the pre-trained RF model to predict the underwater SSP distribution. This approach integrates machine learning with physical modeling to develop

a mesoscale eddy sound speed reconstruction model, as illustrated in Fig.4. The technical route consists of the following steps:

(1) Eddy identification: Using SLA and geostrophic current data derived from satellite altimetry, we applied the Angular Momentum Eddy Detection and Tracking Algorithm (AMEDA) (Le Vu et al., 2018) to systematically process daily observations in the Northwest Pacific Ocean from January 1997 to March 2024. This algorithm enables automated detection and tracking of mesoscale eddies while precisely extracted their key characteristic parameters (including date, eddy boundary contour, eddy center coordinates, radius, and eddy type).

(2) Construction of an eddy-interior thermohaline, density, and sound speed dataset using Argo profiling data: Using the identified eddy features, we perform daily matching between all mesoscale eddies and Argo stations within the study area, selecting Argo profiles observed within eddy-influenced regions during 1997–2024. The density and SSPs are calculated using empirical formulas, ultimately establishing a comprehensive dataset containing temperature, salinity, density, and sound speed characteristics within mesoscale eddies. Concurrently, we extract key eddy parameters (including eddy center position, radius, and type) along with their corresponding sea surface environmental parameters (SST, SLA, SSS, and SSD).

(3) Learning the mapping relationship from surface parameters to underwater sound speed: The sea surface environmental parameters (SST, SLA, SSS, and SSD) and Argo density data serve as inputs for SSP reconstruction. RF is used to develop the “surface-to-underwater” sound speed mapping relationship.

(4) Density reconstruction with the mesoscale eddy unified structural model: The density profile in closest proximity to the eddy center is employed to reconstruct the density anomaly for all profiles within the eddy, in accordance with mesoscale eddy unified structural theory. These anomalies are subsequently amalgamated with the climatological mean density (ARMOR 3D) to ascertain the reconstructed mesoscale eddy density field.

(5) The sea surface environmental parameters and the reconstructed mesoscale eddy density are entered into the mapping relationship to generate the SSP within the eddy.

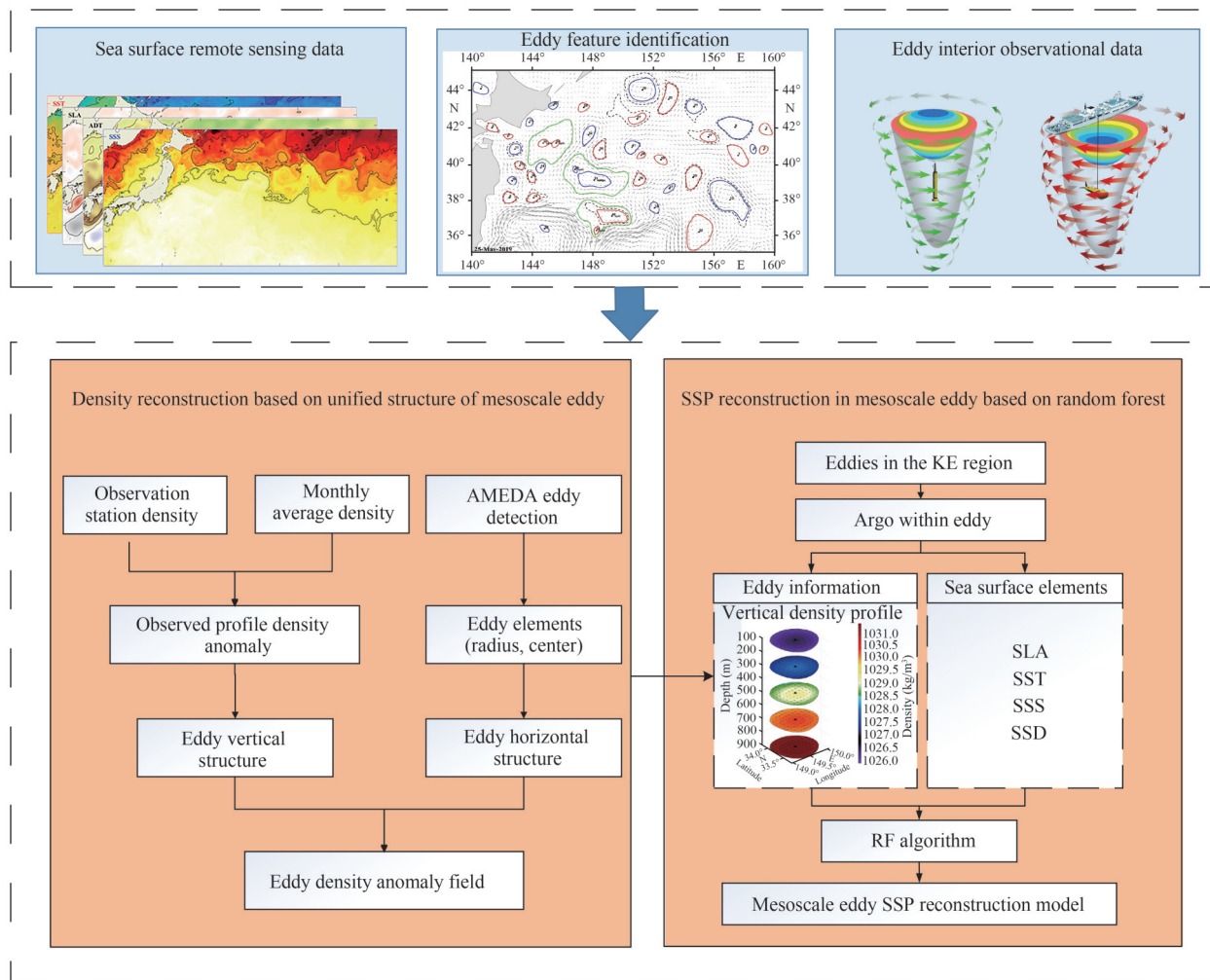


Fig.4 The technical process of PIRF-DEN

4 EXPERIMENT ANALYSIS

4.1 Data preprocessing

The present study focuses on the Kuroshio Extension region (130°E – 180°E , 25°N – 45°N), where sea surface environmental data were spatiotemporally matched with Argo scatter points within mesoscale eddies. This matching was achieved by aligning satellite data daily with Argo observations in the time domain, while employing bilinear interpolation to map gridded satellite data onto Argo profile locations in the spatial domain. All Argo profiles were then interpolated to uniform depth levels (0–1 000 m, with 10-m intervals, totaling 101 layers) to establish a consistent mesoscale eddy Argo profile dataset for subsequent analysis. The identification of a total of 38 193 profiles within mesoscale eddies was conducted from May 1998 to March 2024. Due to limitations in Argo data quality, a series of quality

control procedures were applied to the selected eddy data. The specific steps are as follows:

(1) Removal of invalid profiles: older Argo salinity data often contain invalid values. Profiles with invalid salinity data were removed.

(2) Outlier detection: given the extreme conditions in the Northwest Pacific, Argo profiles with temperature values outside the range of -2.5 – 32°C or salinity values outside the range of 30–40 were excluded.

(3) Climatological screening: a comprehensive quality control protocol was implemented to ensure the integrity of the profile selection process. Vertical profiles exhibiting temperature anomalies greater than 2°C or salinity deviations exceeding 0.2 relative to the climatological mean were classified as statistically significant outliers and systematically excluded from the training dataset.

Subsequent to the implementation of the

aforementioned quality control procedures, the number of Argo profiles within mesoscale eddies was diminished from 38 193 to 29 329. These 29 329 Argo density and SSPs collectively constitute the mesoscale eddy Argo temperature-salinity-sound speed dataset for the Kuroshio Extension region. This comprehensive dataset encompasses a broad spectrum of Argo profiles within mesoscale eddies across the entire Kuroshio Extension region, spanning a considerable temporal range. It also provides ample training samples at various positions along the normalized radius of the mesoscale eddies, ensuring a robust representation of the average state of vortices in this region.

4.2 SSP reconstruction modeling process

The PIRF-DEN model employs the RF algorithm to establish a nonlinear mapping relationship between sea surface environmental parameters, reconstructed density, and underwater SSPs. In implementing the RF framework, we retained default configurations including bootstrap aggregation, all feature consideration in node splitting, and Gini impurity as the splitting criterion. Through systematic sensitivity analysis, we determined the optimal hyperparameters: 100 trees and 5 minimum leaf samples. The model also incorporated out-of-bag prediction and variable importance evaluation, with parallel computing enabled to enhance training efficiency.

For the station-by-station SSP reconstruction methods based on the EOF method (sEOF-r method and sEOF-RF method), the critical aspect is selecting the appropriate EOF modal order for sound speed. The selection of the number of EOF modes should not rely solely on a fixed variance contribution threshold but should also take into account the physical interpretability of the modes. North et al. (1982) pointed out that if the error ranges of the eigenvalues of adjacent EOF modes overlap, they become physically indistinguishable, and higher-order modes may reflect noise rather than true physical processes. Although including these modes may increase the variance contribution, it can also introduce noise, increase model complexity, and raise the risk of overfitting.

The mesoscale eddy sound speed profile dataset constructed in this study is strongly influenced by physical processes. As shown in Table 2, the first mode already accounts for 91.76% of the variance, sufficiently capturing the core variability of the SSPs in the region. Starting from the second mode, the spatial structure becomes more complex, the

physical meaning is less clear, and the eigenvalues decrease rapidly, exhibiting characteristics typical of “noise modes.” Therefore, following the approach of Chen et al. (2018a), we ultimately decided to use only the first EOF mode for the reconstruction of mesoscale eddy SSPs, ensuring both model reliability and simplicity.

4.3 Performance test of the PIRF-DEN model reconstruction

The PIRF-DEN model was evaluated using a mesoscale eddy Argo temperature-salinity-density-sound speed dataset and remote sensing data, with sea surface environmental parameters (SST, SLA, SSS, and SSD) and Argo-derived density serving as inputs. The dataset was split into a 90% training set and a 10% testing set. The reconstruction performance was then assessed and compared to that of three alternative models: sEOF-r, sEOF-RF, and RF. As illustrated in Fig. 5, the MAE and RMSE of sound speed reconstruction at different depths were calculated based on the test set. Overall, the PIRF-DEN model and the RF method outperformed the EOF-based models (sEOF-RF and sEOF-r). The

Table 2 Variance contribution rates of the first five modes

Mode number	1	2	3	4	5
Variance contribution rate (%)	91.76	4.97	1.81	0.76	0.29

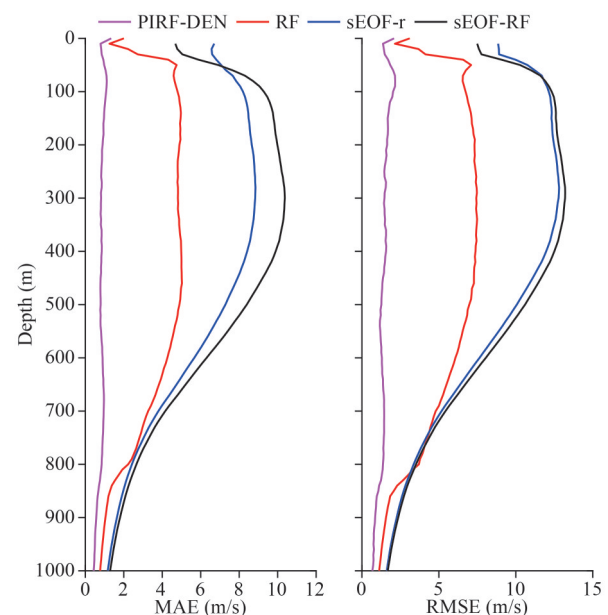


Fig. 5 Mean absolute error (MAE) and root mean square error (RMSE) of sound speed reconstruction at various depths for the four SSP reconstruction methods

suboptimal performance of the EOF-based models can be attributed to information loss during the extraction of primary modes for sound speed modeling, which reduces reconstruction accuracy.

In the 0–1 000-m depth range, the PIRF-DEN model substantially enhanced the accuracy of SSP reconstruction, consistently outperforming the other three models at all depths. The most notable improvement was observed at depths shallower than 840 m. Specifically, within the 0–840-m range, the sound speed error for PIRF-DEN is 0.89 m/s. For RF, the error is 4.20 m/s; for sEOF-RF, the error is 6.67 m/s; and for sEOF-r, the error is 7.50 m/s. The PIRF-DEN model demonstrated a substantial enhancement in reconstruction accuracy, with an improvement of approximately 88.1% compared to sEOF-r and an improvement of 78.8% compared to the RF model. These findings underscore the significant advantage of incorporating vertical density information into the PIRF-DEN model. For the 840–1 000-m depth range, the reconstruction errors for all four models decreased to <2 m/s, with the PIRF-DEN model achieving a reconstruction error of <0.7 m/s.

Table 3 lists the average reconstruction accuracy of sound speed within mesoscale eddies for the four models: PIRF-DEN>RF>sEOF-RF>sEOF-r. In comparison to the classical sEOF-r method, the PIRF-DEN model demonstrates an 87.3% enhancement in reconstruction accuracy. The nonlinear framework constructed using a machine learning model (sEOF-RF) exhibited a substantial enhancement in performance compared to the conventional linear framework (sEOF-r), with an improvement in reconstruction accuracy of 11%. Furthermore, the incorporation of vertical density information into the PIRF-DEN model led to a significant enhancement in reconstruction accuracy, with an improvement of 77.28%.

Table 4 summarizes the research progress of various scholars in the field of sound speed reconstruction. The analysis indicates that hybrid models combining the sEOF method with machine learning achieve an average sound speed reconstruction

accuracy of approximately 1–2 m/s in typical oceanic regions. However, in dynamically complex areas such as fronts and mesoscale eddies, the reconstruction error increases significantly. The proposed PIRF-DEN model introduces eddy reconstruction density as an innovative input parameter, effectively improving sound speed reconstruction accuracy in mesoscale eddy regions and constraining errors within 1–2 m/s. Compared to existing methods, this model demonstrates superior performance in sound speed reconstruction.

To further compare the reconstruction performance of the four SSP reconstruction methods, we present the SSP reconstruction errors at all Argo stations for the four models in Fig.6, showing that the reconstruction accuracy of the four models follows the order PIRF-DEN>RF>sEOF-RF>sEOF-r, among them the PIRF-DEN model demonstrated significantly higher accuracy than the other three models. Most stations exhibited sound speed errors below 1 m/s, with only a few stations experiencing errors exceeding 2 m/s, sporadically distributed in the sub-Arctic frontal region. In summary, the PIRF-DEN model proposed in this study can incorporate density as an input through the mesoscale eddy unified structure model, accurately describe the vertical structure of mesoscale eddies, and significantly reduce sound speed reconstruction errors in regions where mesoscale eddies frequently occur.

4.4 Error analysis of sound speed reconstruction based on measured eddies

In the preceding section, the reconstruction performance of the models was evaluated using the test set (Argo profiles). This section further assesses the reconstruction capability of the models based on two observed eddies. Our research team captured

Table 3 Evaluation coefficients of the four SSP reconstruction methods

Evaluation coefficient	SSP reconstruction method			
	PIRF-DEN	RF	sEOF-RF	sEOF-r
MAE	0.83	3.66	5.84	6.56
RMSE	1.39	5.42	8.28	8.51

Table 4 The related work of other scholars

Reference	Year	SSP reconstruction method	RMSE (m/s)	Study region
Li et al., 2022	2022	sEOF-SOM	1.69	Southeastern Indian Ocean
		sEOF-MLR	1.63	South China Sea
Feng et al., 2024	2024	sEOF-SVR	1.53	South China Sea
		sEOF-XGBoost	1.16	South China Sea
Zhao et al., 2024	2024	sEOF-LSTM	1.76	South China Sea
Liu et al., 2024b	2024	sEOF-GRNN	1.50	Luzon Strait

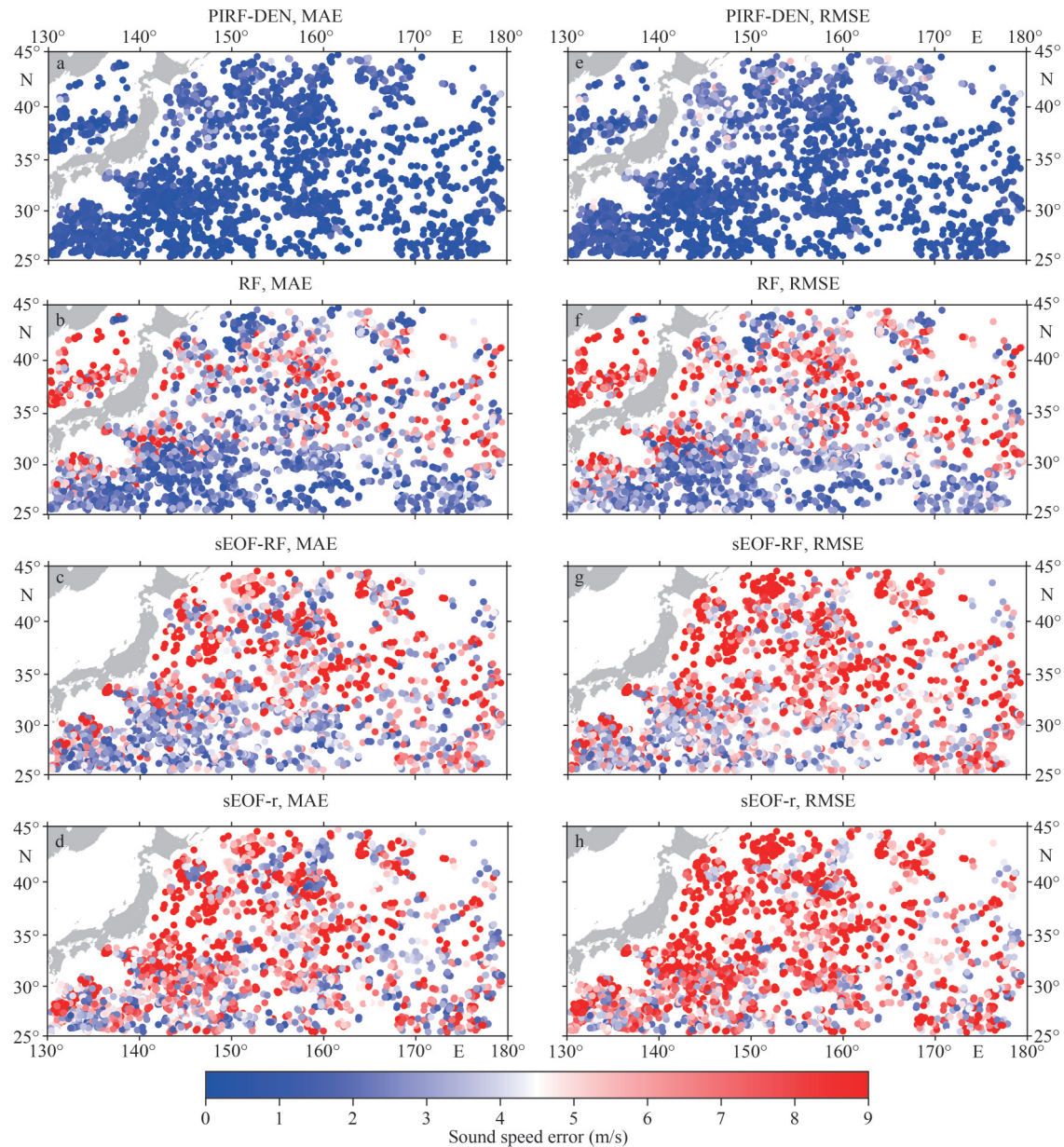


Fig.6 Sound speed errors at all Argo stations for the four SSP reconstruction methods

and observed two mesoscale eddies in the Kuroshio Extension region, with detailed information on these eddies listed in Table 5. Eddy 1 is an anticyclonic eddy, and the observational data originate from the Pacific Mode Water Ventilation Experiment (P-MoVE) conducted in March–April 2014 using Argo. During the experiment, 17 Argo floats were

deployed inside the anticyclonic eddy (Fig.7a), which collected high-frequency temperature and salinity profiles daily. For the present study, 15 Argo profiles observed on March 29, 2014, within the eddy were selected for sound speed reconstruction and comparison. Eddy 2 is a cyclonic eddy, and the observational data were collected on June 15, 2019

Table 5 Observed eddy information

Eddy No.	Eddy type	Eddy center coordinate	Radius (km)	Instrument	Observation date	Number of stations
1	Anticyclonic	29.625°N, 147.168°E	101.38	Argo	2014.3.29	15
2	Cyclonic	33.708 3°N, 149.541 7°E	51.81	MVP	2019.6.15	24

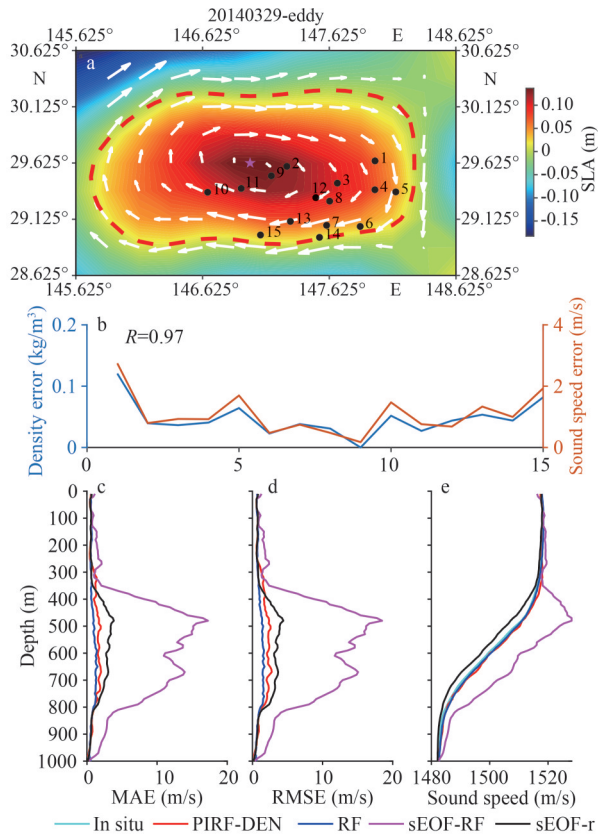


Fig.7 Overview of the SSP reconstruction performance for Eddy 1

a. illustration of Eddy 1 and the Argo stations within it, where the red line represents the eddy's edge, the pentagram indicates the eddy center, and the black dots denote the Argo observation stations; b. the correlation between density reconstruction error and SSP reconstruction error; c. distribution of sound speed MAE; d. distribution of sound speed RMSE; e. comparison of the average reconstructed SSPs from the four methods with the average SSP derived using the empirical formula.

(Fig.8a), using an MVP (moving vessel profiler) during a ship-based continuous survey of the Northwest Pacific. The comprehensive survey encompassed a total of 24 observation stations, thereby forming a complete cross-section within the eddy.

The PIRF-DEN model constructed in this study was utilized to reconstruct the density profiles for all observation stations within the eddy for each observed eddy. The density profiles at these stations were reconstructed based on measured profiles near the eddy center. The sea surface parameters and reconstructed density profiles were then entered into the PIRF-DEN model to generate the SSPs for the respective stations. The SSP reconstruction process for the other three methods followed the same procedures described earlier. The reconstruction performance for each method is shown in Table 6.

Table 6 Reconstruction performance of the four SSP reconstruction methods for observed eddies

Reconstruction method	MAE (m/s)			RMSE (m/s)		
	Eddy 1	Eddy 2	Average	Eddy 1	Eddy 2	Average
PIRF-DEN	1.06	2.53	1.79	1.39	3.13	2.26
RF	0.80	4.05	2.43	1.01	4.73	2.87
sEOF-r	1.50	4.11	2.80	1.80	5.25	3.53
sEOF-RF	6.10	2.63	4.36	6.62	3.61	5.13

For Eddy 1 (Fig.7), the profile observed at Station 9, which is closest to the eddy center, was selected as the constrained profile. Utilizing the unified mesoscale eddy structure model, the density profiles of the remaining stations within the eddy were reconstructed. The sea surface information and the reconstructed density profiles corresponding to the stations were then used as inputs to the model, thereby producing the reconstructed SSP within the eddy. As shown in Table 6, the PIRF-DEN model achieved an average MAE of 1.06 m/s for the reconstructed SSP, slightly less accurate than the RF method's 0.80 m/s but superior to the sEOF-r and sEOF-RF methods, with a 29.3% improvement in accuracy compared to the sEOF-r method. As illustrated in Fig.7c–e, all four methods demonstrate substantial errors in the 350–800-m depth range. The sEOF-RF method exhibits overfitting, resulting in notably higher reconstruction errors for sound speed compared to the other three methods.

Figure 7b further presents the correlation analysis between density reconstruction errors and sound speed reconstruction errors at various stations within Eddy 1. A strong positive correlation ($R=0.97$) is clearly observed between these two error types in the PIRF-DEN model, statistically confirming the direct influence mechanism of density field reconstruction accuracy on sound speed reconstruction results. It should be particularly noted that in this case study, both training and testing datasets were derived from Argo profile observations, where the “surface-to-underwater” sound speed mapping relationship exhibits strong coupling characteristics. Under these conditions, even minor density reconstruction errors can be amplified and propagated to the sound speed reconstruction results, which likely explains why the PIRF-DEN model shows slightly inferior performance in sound speed reconstruction accuracy compared to the RF model.

For Eddy 2 (Fig.8a), the profile observed at Station 12, which is closest to the eddy center, was

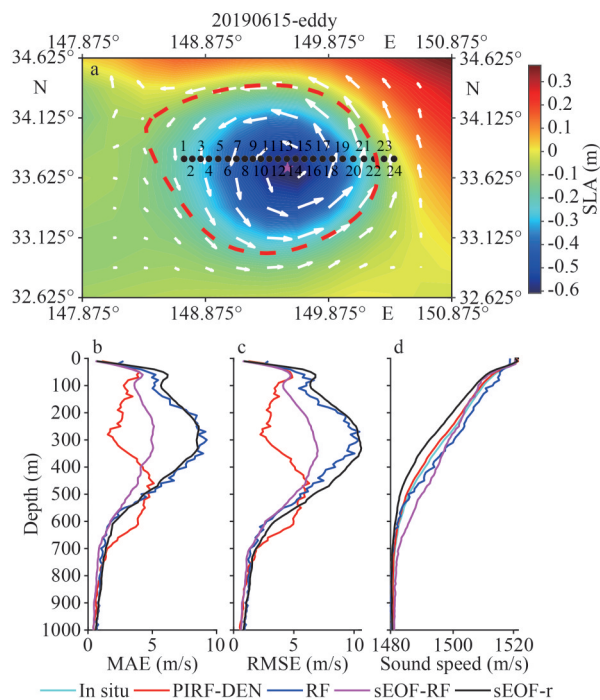


Fig.8 Overview of SSP reconstruction results for Eddy 2
(a) and the meanings of panels (b–d) are similar to those in Fig.7

selected as the constrained profile for density reconstruction. According to Table 6, the reconstruction accuracy of the four models follows this order: PIRF-DEN>RF>sEOF-RF>sEOF-r. The PIRF-DEN model demonstrated an average MAE of 2.53 m/s in sound speed reconstruction, enhancing the accuracy by 38.5% compared to the sEOF-r approach.

As demonstrated in Fig.8b–d, the sound speed reconstruction errors for all four methods are predominantly concentrated between 80 and 600 m. The PIRF-DEN model demonstrates the most substantial enhancement in the 100–400-m range but exhibits slightly suboptimal performance in comparison to the other three models in the 500–700-m range.

Figure 9a further presents the correlation analysis between density reconstruction errors and sound speed reconstruction errors at various stations within Eddy 2. Similarly, the sound speed errors reconstructed by the PIRF-DEN model show a strong positive correlation with density reconstruction errors, with a correlation coefficient of 0.97. For Eddy 2, the training and testing datasets are derived from profiles measured by different CTD instruments, which exhibited distinct sampling frequencies, methodologies, and vertical resolutions. Consequently, the data quality of MVP measurements significantly

differed from that of Argo data, resulting in notably larger sound speed reconstruction errors compared to those observed for Eddy 1. Given that the profile closest to the eddy center was utilized as the constraint for reconstructing the entire section, it stands to reason that the density reconstruction accuracy is inherently higher at the eddy center (the region between the two black dashed lines in Fig.9a, i.e., Stations 7–17) than at the eddy edges. This observation is in alignment with the statistical results presented in Fig.9a.

As illustrated in Fig.9b–e, the sound speed reconstruction errors across the section are concentrated in the upper ocean (100–600 m) for all four models. However, the spatial distribution of high-error regions differs among the models. The PIRF-DEN model, in particular, demonstrates a concentration of errors at the eddy edges, exhibiting an average MAE of 3.48 m/s at these locations. This is notably higher than the 1.40-m/s MAE observed at the eddy center, thereby substantiating the statistical outcomes depicted in Fig.9a. In contrast, the RF model demonstrates the highest reconstruction errors at the eddy center and outside the eddy (Stations 23–24), indicating that reliance on sea surface environmental data alone is inadequate for the precise depiction of the sound speed structure at the eddy center. Conversely, the sEOF-RF and sEOF-r methods exhibit elevated error regions primarily at the eddy edges, underscoring the efficacy of incorporating a nonlinear framework in sound speed reconstruction.

In summary, the PIRF-DEN model demonstrates superior performance in sound speed reconstruction, particularly with regard to enhancing accuracy at the eddy center. However, due to limitations in density reconstruction accuracy, the model's capacity to characterize sound speed structures at the eddy edges necessitates further enhancement.

In conclusion, the PIRF-DEN model demonstrates the most stable reconstruction performance, with higher overall accuracy compared to the other three models, irrespective of whether the eddies are cyclonic or anticyclonic. Due to the constraints imposed by vertical density profiles, the reconstructed sound speed is not subject to overfitting. The incorporation of a nonlinear framework into traditional sEOF-r and sEOF-RF methods significantly enhances the model's performance. However, the limited number of observational profiles available within a single eddy may result in overfitting when employing the sEOF-RF method for SSP reconstruction.

It is imperative to acknowledge the implications

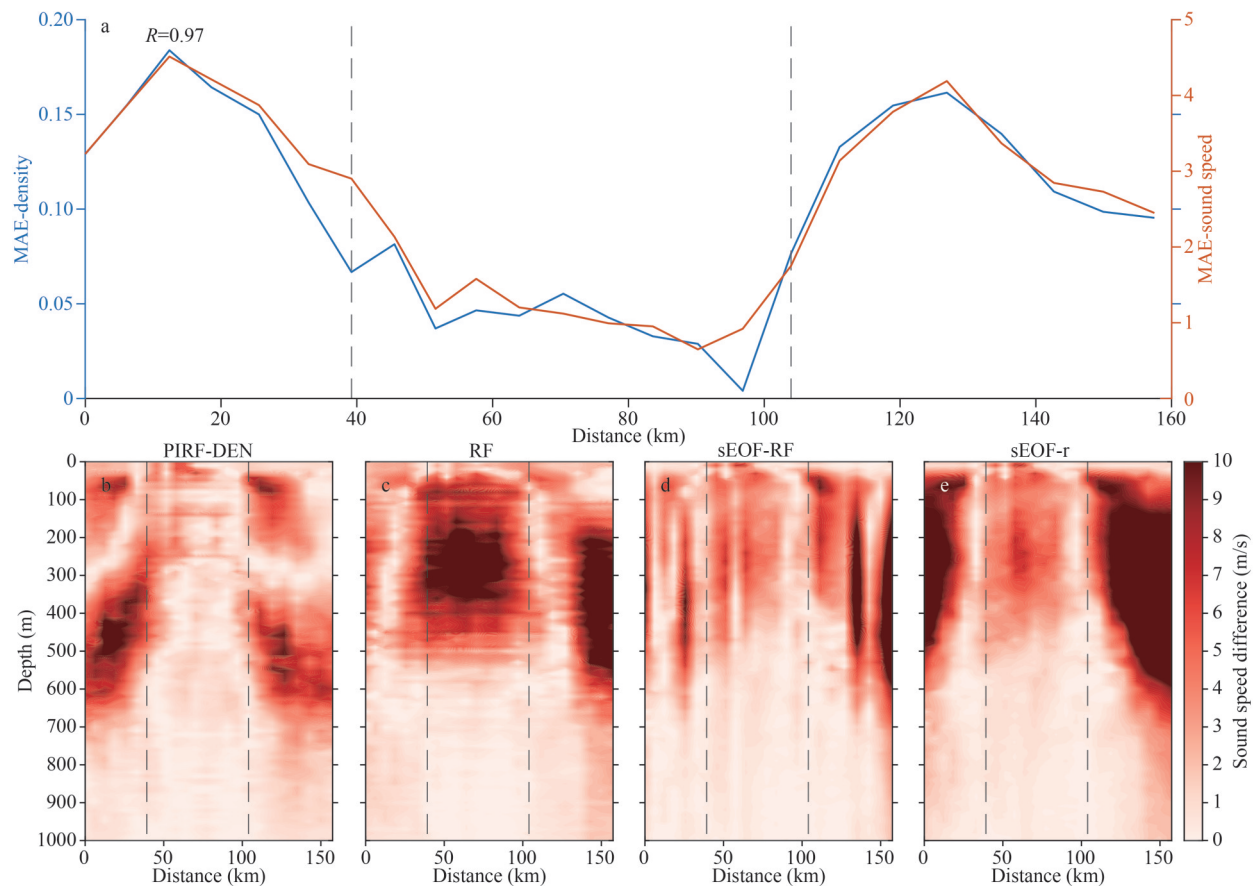


Fig.9 Comparison of sound speed section reconstruction results for Eddy 2

a. variation of density reconstruction errors and sound speed reconstruction errors (MAE) from the PIRF-DEN model with distance. The blue curve represents the density reconstruction error, the orange curve represents the sound speed reconstruction error, and the region between the two black dashed lines indicates the eddy center; b–e. differences between the reconstructed and measured sound speed sections for the PIRF-DEN, RF, sEOF-RF, and sEOF-r methods, respectively. Station 1 is the origin of distance measurements.

of missing data when executing the EOF decomposition on the SSP matrix employing the sEOF method. Specifically, the absence of data for all stations within the designated depth range for EOF decomposition should be avoided. In the event that a station within the eddy exhibits an excessive number of missing data points, it must be regarded as invalid. Conversely, the PIRF-DEN model does not necessitate consideration of missing data for all stations; only a complete observational profile is required for sound speed reconstruction.

5 SOUND PROPAGATION CHARACTERISTICS ANALYSIS

In the preceding section, the validation of the PIRF-DEN model's capacity to enhance the reconstruction accuracy of SSP within an eddy was conducted through the analysis of two observational cases. In the present section, a sound propagation simulation is performed using the Bellhop model,

which is based on the measured and reconstructed sound speeds from Eddy 2. The effectiveness of sound speed reconstruction through sound propagation loss is then analyzed. Given the depth limitation of both measured and reconstructed SSP to 1 000 m, the sound speed distribution below this depth is supplemented using ARMOR 3D climatology data. The parameters for the Bellhop model are as follows: eastward sound propagation (with the sound source at Station 1 and the receiver at Station 24), a horizontal propagation distance of 157 km, a source depth and receiver depth of 100 m each, a source frequency of 200 Hz, a transmission angle ranging from -89° to 89° , and 500 rays. The seabed is characterized by a compressional wave speed of 1 550 m/s, density of 1.5 g/cm³, and attenuation coefficient of 0.13 dB/λ. Boundary conditions include a vacuum sea surface and a semi-infinite half-space seabed.

Figure 10 illustrates a comparison of sound propagation loss for various locations, with both the

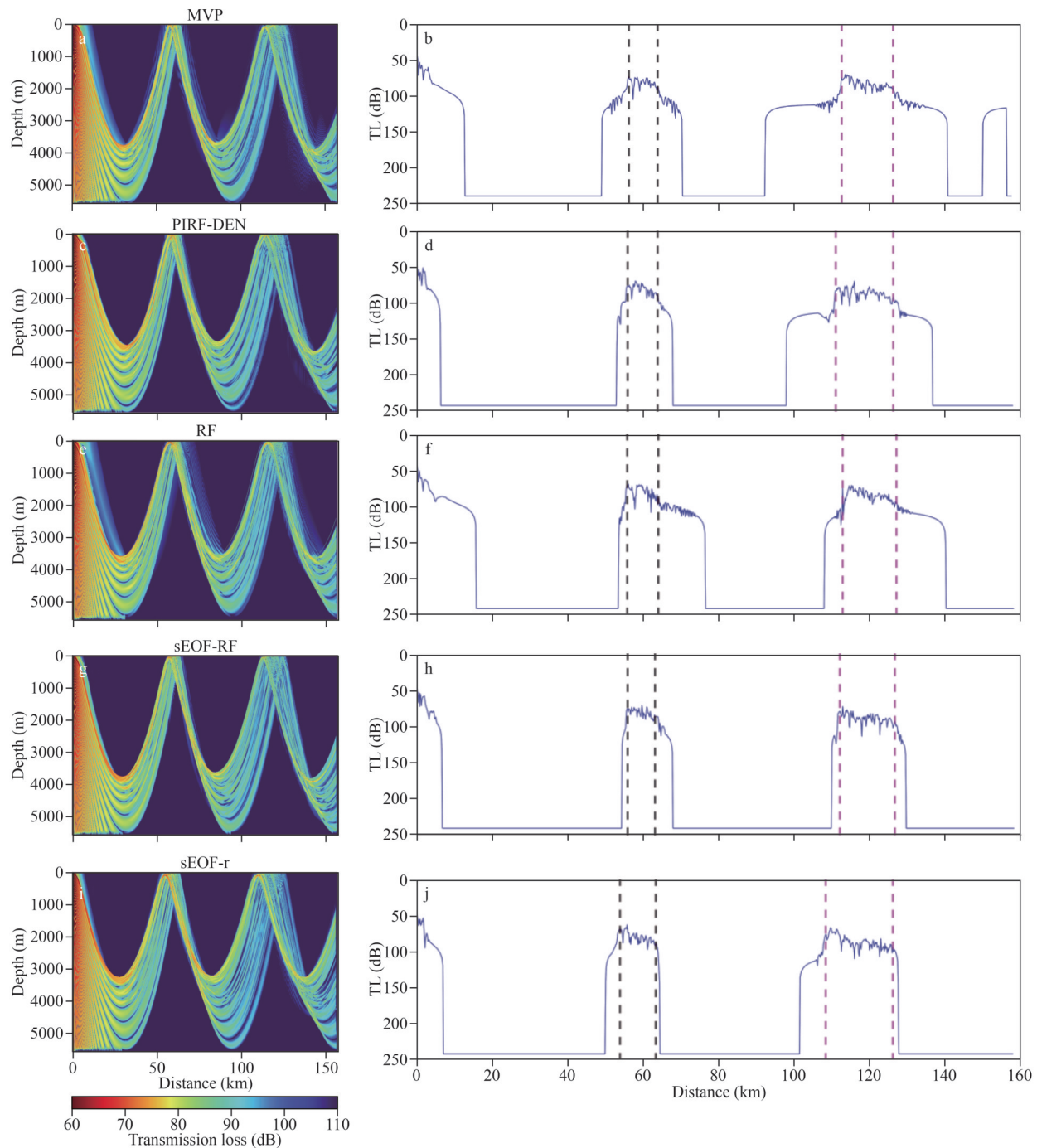


Fig.10 Differences in sound propagation loss calculated using measured and reconstructed sound speeds

Rows 1–5 represent the sound propagation loss calculated based on the measured sound speed (MVP), PIRF model, RF model, sEOF-RF method, and sEOF-r method, respectively. The black dashed line marks the boundary of the first convergence zone, while the magenta dashed line marks the boundary of the second convergence zone.

source and receiver depths fixed at 100 m. The results indicate the presence of two convergence zones within the propagation distance, as calculated based on the four reconstructed sound speeds. However, it is observed that the distance, width, and intensity of these convergence zones differ from the results obtained from measured sound speeds. The

parameters of these convergence zones are enumerated in Table 7. The PIRF-DEN model's sound propagation loss analysis most closely matches the measured results, particularly for the first convergence zone, where the PIRF-DEN model provides the most accurate depiction, with a shift of 315.2 m westward, an increase in width by 315.2 m,

and a 0.44-dB difference in intensity. To assess the consistency of sound propagation losses, we employed correlation analysis. The correlation coefficients for the four models (PIRF-DEN, RF, sEOF-RF, and sEOF-r) are 0.77, 0.67, 0.64, and 0.68, respectively. The highest correlation is observed between the sound propagation loss computed from the MVP sound speed section and the PIRF-DEN sound speed section, indicating that the sound speed profile reconstructed by the PIRF-DEN model exhibits good performance in sound propagation simulation. This result is consistent with the comprehensive analysis in Fig.10 and Table 7.

We observed a notable discrepancy in the transmission loss calculated from MVP data at a range of 150 km compared to other methods. This discrepancy can be attributed to two primary factors:

First, fundamental differences exist in data sources and measurement methodologies. The MVP system employs an underway, pump-free CTD, which differs significantly from the pumped CTD used by Argo floats. MVP measurements are susceptible to sea conditions, vessel motion, and probe-entry disturbances, representing instantaneous point sampling. This is particularly evident in the 100–400-m layer, a critical depth interval characterized by complex hydrographic structures and strong thermohaline gradients, where MVP may fail to accurately capture the regional mean state and could contain unidentified systematic errors. In contrast, reconstruction methods such as PIRF-DEN are trained on a dataset of 29 329 Argo profiles. Argo floats conduct measurements under free-drifting and hydrodynamically stable conditions, providing high-quality, representative data. The

resulting reconstructions thus reflect a statistically smoothed regional climatology. Accordingly, the divergence observed in Fig.10b likely stems from transient fine-scale structures or local perturbations captured by MVP, whereas our model offers a more robust representation of the typical regional state.

Second, the cumulative effect of sound speed profile differences during propagation plays a significant role. As shown in Figs.8b–c and 9b–e, substantial sound speed deviations exist between MVP observations and reconstructed profiles within the 100–400-m layer, which is a key stratum that critically influences acoustic propagation paths and convergence zone formation. When acoustic waves propagate to a range of 150 km, the initial sound speed differences are progressively amplified through range accumulation and multi-path interference effects. Such amplification leads to marked deviations in the position and intensity of convergence zone boundaries. This phenomenon further confirms that acoustic fields are highly sensitive to the vertical structure of SSPs.

6 CONCLUSION

This study focuses on the Kuroshio Extension region to address the issue of large errors in sound speed reconstruction within mesoscale eddies. To construct a dataset of Argo temperature-salinity-density and sound speed data within these eddies, Argo profiles within all eddies in the region from 1997 to 2024 are extracted. Subsequently, a “surface-to-underwater” sound speed mapping relationship was constructed through the use of RF, combining satellite sea surface remote sensing data, the sea surface environmental parameters (SST, SLA, SSS, and SSD), and Argo-derived density.

Table 7 Parameters and deviations of sound convergence zone

Sound field type		First convergence zone			Second convergence zone		
		Distance (km)	Width (km)	TL (dB)	Distance (km)	Width (km)	TL (dB)
Convergence zone parameter	MVP/measured	56.266 7	7.565 3	78.623 5	112.533 4	13.554 4	80.884 2
	PIRF-DEN	55.951 5	7.880 5	78.187 2	110.799 7	15.130 5	83.414 3
	RF	55.636 3	8.195 7	77.559 6	112.375 8	14.184 9	83.171 1
	sEOF-RF	55.793 9	7.722 8	79.399 9	111.588 0	14.500 0	85.737 7
	sEOF-r	53.902 5	9.456 6	77.929 9	108.120 0	17.653 0	84.708 1
Deviations from measured	PIRF-DEN	-0.315 2	+0.315 2	-0.436 3	-1.733 7	+1.576 1	+2.530 1
	RF	-0.630 4	+0.630 4	-1.063 9	-0.157 6	+0.630 5	+2.286 9
	sEOF-RF	-0.472 8	+0.157 5	+0.776 4	-0.945 4	+0.945 6	+4.853 5
	sEOF-r	-2.364 2	+1.891 3	-0.693 6	-4.413 4	+4.098 6	+3.823 9

Concurrently, a mesoscale eddy unified structural model was applied to reconstruct the density field within the eddy. The application of the mapping relationship, incorporating the sea surface environmental parameters and the reconstructed eddy density, yielded the corresponding SSP of the eddy. The establishment of a mesoscale eddy sound speed reconstruction model integrating machine learning and physical modeling (PIRF-DEN model) was undertaken. The outcomes of comparative experiments led to the following conclusions:

(1) The reconstruction performance test of the PIRF-DEN model indicates that density is the key variable in constraining the vertical structure of mesoscale eddies, and the introduction of density significantly improves the reconstruction accuracy of the SSP within mesoscale eddies. The MAE of the reconstructed sound speed in the PIRF-DEN model is 0.83 m/s, and the RMSE is 1.39 m/s, representing reductions of 87.3% and 83.7%, respectively, compared to the classic sEOF-r method.

(2) The sound speed reconstruction experiment, which is based on observed eddies, demonstrates that the PIRF-DEN model outperforms the other three models in terms of both reconstruction accuracy and stability, irrespective of whether the eddies are cyclonic or anticyclonic. The mesoscale eddy unified structural model effectively characterizes the vertical density structure inside the eddy. Due to the constraint imposed by the vertical density profile, the sound speed reconstruction is less prone to overfitting.

(3) In the context of sound propagation simulations, the loss of sound energy, calculated based on the measured and reconstructed SSPs, exhibits a high degree of consistency, as evidenced by a correlation coefficient exceeding 0.75. This finding further substantiates the reliability of the sound speed profile reconstructed by the PIRF-DEN model.

7 DATA AVAILABILITY STATEMENT

All data generated and analyzed during this study are available from the corresponding author upon reasonable request.

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